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THE

MATHEMATICAL MONTHLY.

Vol. II. ... FEBRUARY, 1860. ... No. V.

PRIZE PROBLEMS FOR STUDENTS.

I. Prove that the value of a proper fraction is increased, and an improper fraction diminished, by adding the same quantity to both terms of the fraction; and that the reverse is the case when the same quantity is subtracted from both terms of the fraction.

II. A common tangent is drawn to two circles which touch each other externally; if a circle be described on that part of the tangent which lies between the points of tangency as diameter, this circle will pass through the point of contact of the two circles, and will touch the line joining their centres.

III. Find the n quantities $x_1, x_2, \ldots\ldots x_n$, from the n equations,

$$x_1 + x_2 + x_3 \ldots\ldots + x_n = A,$$
$$a_1 x_1 + a_2 x_2 + a_3 x_3 \ldots\ldots + a_n x_n = 0,$$
$$a_1^2 x_1 + a_2^2 x_2 + a_3^2 x_3^2 \ldots\ldots + a_n^2 x_n = 0,$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$a_1^{n-1} x_1 + a_2^{n-1} x_2 + a_3^{n-1} x_3 \ldots\ldots + a_n^{n-1} x_n = 0,$$

and obtain symmetrical expressions for x_1, x_2, &c.

IV. Prove that $\sin^n(\theta - \varphi) \sin\varphi$ is a maximum when

$$\sin(\theta - 2\varphi) = \frac{n-1}{n+1} \sin\theta;$$

θ being a given constant, and φ the variable.

V. The notation of Problem V. in the November No. being retained, prove that in the plane triangle

$$\frac{1}{\varrho'}+\frac{1}{\varrho''}+\frac{1}{\varrho'''}=\frac{1}{\varrho};$$

and in the spherical triangle,

$$\frac{1}{\tan\varrho'}+\frac{1}{\tan\varrho''}+\frac{1}{\tan\varrho'''}=\frac{1}{\tan\varrho}\,\frac{\cos(\varrho+\delta)\cos(\varrho-\delta)}{\cos^2 r\cos^2\varrho};$$

$$\frac{\cos\delta'}{\cos\varrho'}+\frac{\cos\delta''}{\cos\varrho''}+\frac{\cos\delta'''}{\cos\varrho'''}=\frac{\cos\delta}{\cos\varrho}.$$

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. II., Vol. II.

The first Prize is awarded to George B. Hicks, Cleveland, Ohio.

The second Prize is awarded to William Hinchcliffe, Barre Plains, Mass.

The third Prize is not awarded.

Prize Solution of Problem III.

By George B. Hicks, Cleveland, Ohio.

Of all right-angled plane triangles having the same given hypothenuse, to find the one whose area is the greatest possible. To be solved by Algebra.

Since the hypothenuse is constant, the area is evidently a maximum when the perpendicular dropped from the right angle upon the hypothenuse is a maximum. Let $x\,y$ be the sides, c the hypothenuse, p the perpendicular, and d one of the segments of the hypothenuse. Then

$$p^2+d^2=x^2,\qquad p^2+(c-d)^2=y^2,\qquad x^2+y^2=c^2;$$

whence

$$(1)\quad d^2-c\,d=-p^2.$$

$$\therefore\ d=\frac{c}{2}\pm\sqrt{\frac{c^2}{4}-p^2}.$$

But the greatest value p^2 can have, and d still be real, is $\frac{1}{4}c^2$; $\therefore p = \frac{1}{2}c$, $d = \frac{1}{2}c$, $\therefore$ the triangle is isosceles, and $x = y = c\sqrt{\frac{1}{2}}$. Otherwise, put $d = w + \frac{1}{2}c$.

$\therefore p^2 = cd - d^2 = \frac{1}{4}c^2 - w^2$, which is obviously a maximum when $w = 0$, $\therefore p = \frac{1}{2}c$ and $x = y$ as before.

Or, (1) may be written $p^2 = d(c - d)$. But $d(c - d)$ is the product of the segments of the hypothenuse, which we know to be a maximum when the segments are equal; the result already found.

Second Solution.

By William Hinchcliffe, Barre Plains, Mass.

Let ABC be right-angled at C, from which drop the perpendicular CD. Put $AB = 2a$, $AD = a + x$, $BD = a - x$. Now, $\frac{1}{2}AB \times CD =$ a maximum. But AB is given; therefore CD and hence $CD^2 =$ a maximum. But $CD^2 = AD \times BD = a^2 - x^2$ = a maximum when $x = 0$. $\therefore AD = BD = CD$. $\therefore AC = BC = a\sqrt{2}$; whence the required triangle will contain the greatest area when the sides containing the right angle are equal.

Prize Solution of Problem IV.

By William Hinchcliffe, Barre Plains, Mass.

What is that fraction, the cube of which being subtracted from it, the remainder is the greatest possible? To be solved by Algebra.

Let x denote the required fraction. Then, by the question, $x - x^3 = y =$ a maximum. Denote a negative root of this equation by $-a$; then $x^3 - x + y = 0$ divided by $x + a = 0$ gives a quotient, $x^2 - ax + a^2 - 1 = 0$ (1), and a remainder, $a^3 - a - y = 0$. But by solving, (1) gives

$$x = \frac{a \pm \sqrt{4 - 3a^2}}{2}.$$

Now, it is evident that, if y is positive, x is less than 1; and since $x - x^3 = y = a^3 - a$, it also follows, that when y is the greatest

possible, a must have the greatest value which will make x real.

$$\therefore\ 4 - 3a^2 = 0;\ \text{or}\ a = \frac{2}{\sqrt{3}}. \qquad \therefore\ x = \frac{1}{\sqrt{3}}.$$

Prize Solution of Problem V.

By George B. Hicks, Cleveland, Ohio.

If, in a plane or spherical triangle, A, B, C denote the angles, and a, b, c the sides respectively opposite them; and if we produce the sides of the triangle, and consider the three circles which touch two of the sides interiorly and the third side exteriorly; and denote by r, ϱ the radii of the circumscribed and inscribed circles; by ϱ', ϱ'', ϱ''', the radii of the circles touching exteriorly the sides a, b, c respectively; by δ', δ'', δ''', the distances of the centres of these circles from the centre of the circle circumscribed about the primitive triangle; then we have in the plane

$$-\sin A + \sin B + \sin C = 4 \cos \tfrac{1}{2} A \sin \tfrac{1}{2} B \sin \tfrac{1}{2} C,$$
$$\sin A - \sin B + \sin C = 4 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} B \sin \tfrac{1}{2} C,$$
$$\sin A + \sin B - \sin C = 4 \sin \tfrac{1}{2} A \sin \tfrac{1}{2} B \cos \tfrac{1}{2} C,$$

and in the sphere

$$-\sin A + \sin B + \sin C = 4 \cos \tfrac{1}{2} A \sin \tfrac{1}{2} B \sin \tfrac{1}{2} C \cdot \frac{\cos \delta'}{\cos r \cos \rho'},$$
$$\sin A - \sin B + \sin C = 4 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} B \sin \tfrac{1}{2} C \cdot \frac{\cos \delta''}{\cos r \cos \rho''},$$
$$\sin A + \sin B - \sin C = 4 \sin \tfrac{1}{2} A \sin \tfrac{1}{2} B \cos \tfrac{1}{2} C \cdot \frac{\delta'''}{\cos r \cos \rho'''}.$$

First Part. — In plane trigonometry

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s-c)(s-b)}{bc}}, \qquad \cos \tfrac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}};$$

and similar expressions may be found for the other angles by simply permuting the letters. Now

$$-\sin A + \sin B + \sin C = 2(-\sin \tfrac{1}{2} A \cos \tfrac{1}{2} A + \sin \tfrac{1}{2} B \cos \tfrac{1}{2} B + \sin \tfrac{1}{2} C \cos \tfrac{1}{2} C)$$
$$= 2M\left(\frac{-a+b+c}{abc}\right),$$

putting $M = \sqrt{s(s-a)(s-b)(s-c)}$. But

$$4 \cos \tfrac{1}{2} A \sin \tfrac{1}{2} B \sin \tfrac{1}{2} C = 2M\left(\frac{-a+b+c}{abc}\right),$$

observing that $s = \frac{1}{2}(a+b+c)$.

$$\therefore\ -\sin A+\sin B+\sin C=4\cos\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C;$$

and in the same way the remaining equations may be found.

Second Part. — By a process precisely similar to the above we shall find for the spherical triangle

$$-\sin A+\sin B+\sin C=2M\left(\frac{-\sin a+\sin b+\sin c}{\sin a\sin b\sin c}\right),$$

$$4\cos\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C=4M\left(\frac{\sin(s-a)}{\sin a\sin b\sin c}\right).$$

It only remains to show that

$$\frac{\cos\delta'}{\cos r\cos\varrho'}=\frac{-\sin a+\sin b+\sin c}{2\sin(s-a)}.$$

Let ABC be any spherical triangle, and O the centre of its circumscribed circle. Produce AB and AC until they meet in A'. Let $A'C=b_1$, and $A'B=c_1$. Put $BK=\alpha$, $KA'=\beta$, $CH=\gamma$. Then $a=\alpha+\gamma$, $b_1=\beta+\gamma$, $c_1=\alpha+\beta$.

$$\therefore BK=\alpha=\frac{a-b_1+c_1}{2}=\frac{a-(\pi-b)+(\pi-c)}{2}$$

$$=\frac{a+b-c}{2}=s-c.$$

We also have $BO=r$, $O'K=\varrho'$, $OO'=\delta'$; and since AOB, AOC, BOC are isosceles, $OBC=\frac{1}{2}(-A+B+C)$, $CBO'=\frac{1}{2}(\pi-B)$;

$\therefore OBO'=\frac{1}{2}\pi-\frac{1}{2}(A-C)$. $\therefore \cos OBO'=\sin\frac{1}{2}(A-C)=\dfrac{\cos\delta'-\cos r\cos BO'}{\sin r\sin BO'}$.

But from the right-angled triangle $BO'K$,

$$\cos BO'=\cos\varrho'\cos(s-c),\quad \sin BO'=\frac{\sin\varrho'}{\sin\frac{1}{2}(\pi-B)}=\frac{\sin\varrho'}{\cos\frac{1}{2}B}.$$

Substituting the values of $\cos BO'$, $\sin BO'$, in that of $\sin\frac{1}{2}(A-C)$, we get, by observing that $\dfrac{\sin\frac{1}{2}(A-C)}{\cos\frac{1}{2}B}=\dfrac{\sin\frac{1}{2}(a-c)}{\sin\frac{1}{2}b}$,

$$\frac{\cos\delta'}{\cos r\cos\varrho'}=\frac{\sin\frac{1}{2}(a-c)}{\sin\frac{1}{2}b}\tan r\tan\varrho'+\cos(s-c).$$

But by YOUNG's Trig. pp. 128, 134,

$$\tan r \tan \varrho' = \frac{2 \sin \frac{1}{2} a \sin \frac{1}{2} b \sin \frac{1}{2} c}{\sin (s - a)};$$

$$\therefore \frac{\cos \delta'}{\cos r \cos \varrho'} = \frac{2 \sin \frac{1}{2} a \sin \frac{1}{2} c \sin \frac{1}{2} (a - c) + \sin (s - a) \cos (s - c)}{\sin (s - a)}$$

$$= \frac{-\sin a + \sin b + \sin c}{2 \sin (s - a)}, \text{ as required.}$$

The value of $\tan r$ and $\tan \varrho'$ is easily found from the triangles $B\,O\,G$, $B\,O'\,K$, and $\tan r$ may be reduced to an expression in terms of the sides by making use of the usual formulas, which give $\sin \frac{1}{2} A$, $\cos \frac{1}{2} A$, &c. in terms of the sides, and $\sin \frac{1}{2} a$, $\cos \frac{1}{2} a$, &c. in terms of the angles. Whence will result the value of $\tan r \tan \varrho'$ given above.

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PRIZE ESSAY ON CENTRAL FORCES.

By DAVID TROWBRIDGE, Perry City, Schuyler Co., N. Y.

1. IN the following essay I shall take the well-known and fundamental equations of motion

(1) $D_t^2 x + X = 0, \quad D_t^2 y + Y = 0, \quad D_t^2 z + Z = 0,$

for granted, as they are to be found in treatises on Mechanics. In these equations x, y, z are the rectangular co-ordinates of the moving body, and X, Y, Z are the forces directed along the co-ordinate axes of x, y, z respectively.

2. Let R represent the central force acting on the moving body, M; let α, β, γ be the angles which its direction makes with the co-ordinate axes x, y, z respectively; let r equal the distance which

separates the centre of gravity of the body M from the centre of gravity of the system, this latter point being the centre of attraction. We then have, assuming this centre for the origin of co-ordinates,

$$(2) \qquad X = R\cos\alpha, \quad Y = R\cos\beta, \quad Z = R\cos\gamma.$$

$$(3) \qquad x = r\cos\alpha, \quad y = r\cos\beta, \quad z = r\cos\gamma.$$

Equations (2) give

$$(4) \qquad \frac{X}{Y} = \frac{\cos\alpha}{\cos\beta}, \quad \frac{Y}{Z} = \frac{\cos\beta}{\cos\gamma}, \quad \frac{Z}{X} = \frac{\cos\gamma}{\cos\alpha}.$$

Equations (3) give

$$(5) \qquad \frac{x}{y} = \frac{\cos\alpha}{\cos\beta}, \quad \frac{y}{z} = \frac{\cos\beta}{\cos\gamma}, \quad \frac{z}{x} = \frac{\cos\gamma}{\cos\alpha}.$$

Substituting these values in equations (4), clearing of fractions, and transposing into the first member, we obtain

$$(6) \qquad Xy - Yx = 0, \quad Yz - Zy = 0, \quad Zx - Xz = 0.$$

Substituting in these equations the values of $X\,Y\,Z$, as given by equations (1), we obtain

$$(7) \quad y\,D_t^2 x - x\,D_t^2 y = 0, \; z\,D_t^2 y - y\,D_t^2 z = 0, \; x\,D_t^2 z - z\,D_t^2 x = 0.$$

3. The integrals of (7) give, letting c, c', and c'' be the arbitrary constants introduced by integration,

$$(8) \quad y\,D_t x - x\,D_t y = c, \quad z\,D_t y - y\,D_t z = c', \quad x\,D_t z - z\,D_t x = c''.$$

If we multiply each of these equations by the variable which it does not contain, and take their sum, it will reduce to

$$(9) \qquad c'x + c''y + cz = 0.$$

This is the equation of a plane passing through the origin of co-ordinates, or centre of attraction; and hence the co-ordinates of the body M are also the co-ordinates of a plane; and therefore the curve described by M is always found in the same plane, or it is a

plane curve. It will not be necessary, then, to retain z in our discussion of the subject; and by putting it equal to nothing our investigation will be much simplified, and the conclusions none the less general.

4. Putting z and Z equal to nothing in equations (1) and (8), we obtain

(10) $D_t^2 x + X = 0$, (11) $D_t^2 y + Y = 0$, (12) $y D_t x - x D_t y = c$.

The integral of (12) gives

(13) $$\int (y\,dx - x\,dy) = ct + C.$$

5. Let us now interpret equation (13). In Fig. 1 let PMY represent the curve described by the body M; S, the centre of attraction; SY and SP, the axes of y and x respectively; $PSM = \varphi =$ the angle which the radius-vector makes with the axis of x, and the other lines as marked in the figure. Then the area $PXM = \int y\,dx'$. But $x' = SP - x$; and since SP is constant, we have $dx' = -dx$. Substituting this value in the above equation, we find the area $PXM = -\int y\,dx$. The sector PSM is made up of the area PXM and the triangle SMX. Therefore sector $PSM = SMX + PXM = \frac{1}{2}xy + \int y\,dx' = \frac{1}{2}xy - \int y\,dx$. This being differentiated and reduced, gives

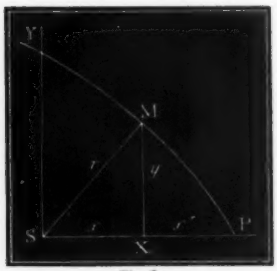

Fig. 1.

$$d(\text{sector } PSM) = -\frac{y\,dx - x\,dy}{2} = -\tfrac{1}{2}c\,dt, \text{ from (12).}$$

We therefore have

(14) $$2 \text{ sector } PSM = -\int (y\,dx - x\,dy) = -ct + C.$$

Had we chosen the sector SYM, we should have found it equal to $+\int (y\,dx - x\,dy)$, the only difference being in the sign of the expression. In the first case the area is reckoned from the axis SP, and in the second it is reckoned from the axis SY. We therefore

infer that equation (13) is an expression for the area passed over by the radius-vector. This area ought to be equal to nothing when $t = 0$; hence $C = 0$. This reduces (13) to

$$f(y\,dx - x\,dy) = c\,t. \tag{15}$$

Since c is constant (15) makes known this law; *The area passed over by the radius-vector is proportional to the time.*

6. This was found to hold true in the motion of the planet Mars, by the celebrated JOHN KEPLER; and he extended it by analogy to all the planets. It has since been called KEPLER'S *second law.* This law admits of a generalization wholly unknown to its discoverer; since it is altogether independent of the form of the curve described, and of the nature of the force, either attractive or repulsive. All that is necessary is, that the force be directed along a single line, and that line the one which connects the centres of gravity of the central and moving bodies.

7. To determine the constant c, let $t = 1$ in equation (15), and we find c equal to twice the area passed over in the unit of time.

8. By referring to Fig. 1 we easily find

$$x = r\cos\varphi, \quad y = r\sin\varphi. \tag{16}$$

Differentiating with respect to the time, t, we obtain

$$D_t x = D_t r\cos\varphi - r\sin\varphi\, D_t\varphi, \quad D_t y = D_t r\sin\varphi + r\cos\varphi\, D_t\varphi. \tag{17}$$

Substituting these values in equation (12) it reduces to

$$r^2 D_t\varphi = -c. \tag{18}$$

Or changing the axis from which the area is reckoned (Art. 5) we have

$$r^2 D_t\varphi = c, \tag{19}$$

$$D_t\varphi = \frac{c}{r^2}. \tag{20}$$

Since φ is the space described at the unit of distance, it follows that $D_t\varphi = \omega$, is the velocity at the unit of distance, or it is the angular velocity of the body M. We hence see that (20) makes known this

law, namely, *The angular velocity varies inversely as the square of the distance.* According to NEWTON, the force of gravity in the solar system varies inversely as the square of the distance; hence, in the case of nature, *the force of gravity varies directly as the angular velocity.* But equation (20) comprehends more, for it is as general as the law given by (15), from which it was derived.

9. Equation (19) can be used in computing the time when the angle is given, and *vice versa*, knowing the form of the orbit.

(21) $t = \frac{1}{c}\int r^2\, d\varphi,$ (22) $\varphi = c\int \frac{dt}{r^2}.$

If we suppose r^{-2} to be a function of t, as it in general is, we can have $\varphi = c\int f(t)\, dt$ (23), and this can be obtained in the form of a series.

10. To obtain a second integral of equations (10) and (11), multiply the first by $2\,dx$, and the second by $2\,dy$, take their sum, and integrate, and we obtain

(24) $D_t x^2 + D_t y^2 = D_t s^2 = v^2 = C - 2\int(X\,dx + Y\,dy).$

Substitute the values of X and Y derived from equations (2), where $\cos\alpha$ and $\cos\beta$ correspond to $\cos\varphi$ and $\sin\varphi$ in equations (16), and $\cos\varphi = \frac{x}{r}$, $\sin\varphi = \frac{y}{r}$, and we obtain

(25) $D_t x^2 + D_t y^2 = C - 2\int(R.\frac{x}{r}.dx + R.\frac{y}{r}.dy) = C - 2\int R\left(\frac{x\,dx + y\,dy}{r}\right).$

But $r^2 = x^2 + y^2$, and $r\,dr = x\,dx + y\,dy$, which being substituted in (25) gives

(26) $D_t x^2 + D_t y^2 = C - 2\int R\,dr.$

Substituting for $D_t x$ and $D_t y$ their values given by equations (17), we find

(27) $r^2 D_t \varphi^2 + D_t r^2 = C - 2\int R\,dr.$

Equations (18) and (19) both give $r^2 D_t \varphi^2 = \frac{c^2}{r^2}$ and this value substituted in (27) gives

$$\frac{c^2}{r^3} + D_t r^2 = C - 2\int R\,dr. \tag{28}$$

Differentiating and dividing by $2\,dr$, and transposing, we obtain

$$R = \frac{c^2}{r^3} - D_t^2 r. \tag{29}$$

11. To interpret this equation, let us recur to equation (20) which gives

$$\textit{angular velocity} = D_t \varphi = \omega = \frac{c}{r^2}. \tag{30}$$

That force which is exactly equal and directly opposed to the force, which, placed at S, Fig. 1, would cause M to revolve in a circle at the same distance $r = SM$, with the angular velocity at M, is called the *centrifugal force* at M. When the body revolves in a circle, the centrifugal force is equal to the square of the velocity divided by the radius. Since ω denotes the velocity at the unit of distance, ω will represent the velocity at the distance r, in a circle; and consequently *centrifugal force* $= \frac{\omega^2 r^2}{r} = \omega^2 r$. Substituting for ω its value given by (30), and we obtain

$$\textit{centrifugal force} = f = \frac{c^2}{r^3}. \tag{31}$$

This force which we have represented by f, is that part of the value of R in (29) which regulates the angular velocity of the body; for that part of the force which is represented by $D_t^2 r$ acts in the direction of the radius-vector, and cannot affect ω except it be indirectly through its action in increasing and decreasing the length of the radius-vector. Hence, whatever force R exerts that exceeds or falls short of f is expended in changing the distance of M from the centre of attraction. This is termed the *paracentric force;* and the velocity due to it the *paracentric velocity* $(= D_t r)$. When $D_t^2 r$ increases the value of r, it should be affected with the minus sign, and *vice versa*. When $D_t^2 r$ is changing from plus to minus, and con-

versely, it must pass through zero; hence to find that point in a curve where the paracentric force is nothing, take the second differential coefficient of the radius-vector with respect to the time, and put it $= 0$, and the resulting equation will make known the point. As an example take the ellipse whose equation is

$$r = \frac{k^2}{1 + e\cos\varphi}. \tag{32}$$

$$D_t r = D_\varphi r\, D_t\varphi = D_\varphi r \cdot \frac{c}{r^2} = \frac{e\,k^2 \sin\varphi}{(1+e\cos\varphi)^2} \cdot \frac{c}{r^2} = \frac{e\,c\sin\varphi}{k^2}. \tag{33}$$

$$D_t^2 r = D_\varphi (D_t r)\, D_t\varphi = D_\varphi (D_t r) \cdot \frac{c}{r^2} = \frac{e\,c^2\cos\varphi}{k^2 r^2} = 0, \text{ or } \varphi = 90°. \tag{34}$$

And (33) gives for the paracentric velocity at the point, $\frac{e\,c}{k^2}$.

12. Equation (29) gives, since $D_t^2 r = D_\varphi^2 r\, D_t\varphi^2 = \frac{c^2}{r^4} \cdot D_\varphi^2 r$,

$$R = \frac{c^2}{r^4}(r - D_\varphi^2 r). \tag{35}$$

If we let $r = u^{-1}$ (36), we have $D_t r = D_\varphi r\, D_t\varphi = \frac{c\,D_\varphi r}{r^2} = -c\,D_\varphi u$.

This value of $D_t r$ substituted in (28) gives, since $dr = -\frac{du}{u^2}$,

$$c^2(u^2 + D_\varphi u^2) = C + 2\int^{R\,du}_{u^2}. \tag{37}$$

Differentiating and dividing by $2\,du$, we find

$$R = c^2 u^2 (u + D_\varphi^2 u). \tag{38}$$

Equations (35) and (38) are useful for finding the law of the force when the equation of the curve is given. As an example, let the given curve be a common parabola, whose equation is

$$\frac{1}{r} = u = \frac{1+\cos\varphi}{2q}, \quad D_\varphi^2 u = -\frac{\cos\varphi}{2q} \text{ and } u + D_\varphi^2 u = \frac{1}{2q}. \tag{39}$$

Substituting in (38) we obtain

$$R = \frac{c^2 u^2}{2q} = \frac{c^2}{2q\,r^2}. \tag{40}$$

Hence the force R varies inversely as the square of the distance; a problem which NEWTON first solved.

13. To solve the inverse problem, namely, *having given the law of the force to determine the form of the orbit described*, (the solution of which NEWTON has not transmitted to us,) we shall proceed as follows: — Suppose R to vary according to NEWTON's hypothesis, viz.: directly as the mass and inversely as the square of the distance; we then have $R = \frac{m}{r^2}$, (41). Let $r = 1$, and $R = m =$ the force at the unit of distance. This value of R substituted in (26) and (28) gives

$$D_t x^2 + D_t y^2 = D_t s^2 = v^2 = C + \frac{2m}{r}, \tag{42}$$

$$\frac{c^2}{r^2} + D_t r^2 = C + \frac{2m}{r}. \tag{43}$$

But $D_t r = D_\varphi r \; D_t \varphi = D_\varphi r \cdot \frac{c}{r^2}$, and hence, by putting $C = h$

$$\frac{c^2}{r^4}(r^2 + D_\varphi r^2) = h + \frac{2m}{r}. \tag{44}$$

Reducing this equation, and we find

$$c\, D_\varphi r = r\sqrt{h r^2 + 2mr - c^2}. \tag{45}$$

To facilitate the integration, put

$$r = \frac{c}{p + \frac{m}{c}}. \tag{46}$$

This reduces (45) to

$$c\, D_\varphi p = -\sqrt{h c^2 + m^2 - c^2 p^2}. \tag{47}$$

Inverting and integrating,

$$\varphi + \theta = \int \frac{-c\,dp}{\sqrt{h c^2 + m^2 - c^2 p^2}} = \cos^{-1} \frac{cp}{\sqrt{h c^2 + m^2}}, \tag{48}$$

θ being a constant. This equation gives

$$cp = \frac{c^2}{r} - m = \sqrt{h c^2 + m^2}\, \cos(\varphi + \theta). \tag{49}$$

14. In order to determine what curve is described, we shall put (49) under the form

$$(50)\quad r = \frac{c^2}{m + \sqrt{h\,c^2 + m^2}\cos(\varphi + \theta)} = \frac{\frac{c^2}{m}}{1 + \sqrt{\frac{h\,c^2}{m^2} + 1}\cos(\varphi + \theta)}.$$

Equation (50) is of the form of the general equation of the conic sections, referred to the focus as origin, from which we infer that the centre of attraction is at the focus, and it gives

$$(51)\quad \frac{c^2}{m} = p = \text{semi-parameter.}\ \sqrt{\frac{h\,c^2}{m^2} + 1} = e = \text{eccentricity.}$$

The factor $\frac{c^2}{m^2}$, being a square, is necessarily positive in the expression $\frac{h\,c^2}{m^2}$; and hence the sign of $\frac{h\,c^2}{m^2}$ will depend upon the sign of h. When h is positive, e will be greater than unity, and the curve described will be a hyperbola; the angle $\varphi + \theta$ being reckoned from the vertex. Had we given the radical in (45) the minus sign, the curve would still be a conic section, but $\varphi + \theta$ would be counted from a different point. When h is negative e will be less than unity, and the curve will be an ellipse; and when $h = 0$, $e = 1$, and the curve is a parabola. When h is negative and equal to $\frac{m^2}{c^2}$, $e = 0$, and the curve is a circle; when h is positive and equal to $\frac{m^2}{c^2}$, $e = \sqrt{2}$, and the curve is an equilateral hyperbola.

15. Let us resume the equation (51), and squaring

$$e^2 = \frac{h\,c^2}{m^2} + 1 = \frac{b^2}{a^2} + 1,$$

for the hyperbola; and $\frac{b}{a} = \frac{c}{m}\sqrt{h}$ (52). We also have $p = \frac{c^2}{m} = -a(1 - e^2)$, and multiplying by a we have

$$\frac{a\,c^2}{m} = -a^2(1 - e^2) = a^2 e^2 - a^2 = b^2,\ \frac{b}{a} = \frac{c^2}{b\,m} = \frac{c}{m}\sqrt{h},$$

by (52) $\therefore\ b = \frac{c}{\sqrt{h}}$ (53), and $a = \frac{m}{h}$ (54).

When h is negative, we have

$$e^2 = 1 - \frac{h c^2}{m^2} = 1 - \frac{b^2}{a^2}, \quad \frac{b}{a} = \frac{c}{m}\sqrt{h},$$

and we consequently have the same expression for the semi-axes as (53) and (54). Since m is constant, the values of the semi-axes will depend upon those of h and c.

16. Let us now determine the value of the constant h. By comparing equation (42) we see that $h = C$,

$$(\text{velocity})^2 = v^2 = h + \frac{2m}{r}.$$

When $r = r_1$, let $v = V$, and we have

$$V^2 = h + \frac{2m}{r_1}, \qquad h = V^2 - \frac{2m}{r_1}. \tag{55}$$

When the curve is a parabola we have seen that $h = 0$, and hence $V^2 = \frac{2m}{r_1}$ (56). This equation gives the velocity in a parabola at any distance r_1 from the centre of attraction, or focus of the curve. When h is positive, we have $V^2 - \frac{2m}{r_1}$ positive, and $V^2 > \frac{2m}{r_1}$. That is, *the velocity in a hyperbola, at the same distance from the centre of attraction, is greater than in the parabola.*

When h is negative, $V^2 - \frac{2m}{r_1}$ is negative, and $V^2 < \frac{2m}{r_1}$. That is, *the velocity in an ellipse, at the same distance from the centre of attraction, is less than in a parabola.*

17. Equation (50) gives the demonstration of KEPLER'S *first law*, viz.: that the planets move in ellipses about the sun, which is situated in one of their foci. Article 16 shows that, had their initial velocities been greater, they would have described either parabolas or hyperbolas.

Let S, Fig. 2, be the centre of attraction, AM the curve described, and SA equal to r. Take the angle $ASM = d\varphi =$ the angle

described in the time dt, and then $AM = ds =$ the space described in the same time. The arc $AB = r\,d\varphi$. But the triangle ABM may be regarded as rectilinear, and right-angled at B; and if we put the angle $AMB = \varepsilon$, we shall have $AB = AM \sin AMB$; or $r\,d\varphi = ds \sin\varepsilon$. Multiplying by r, we have $r^2\,d\varphi = c\,dt = r\,ds \sin\varepsilon$; or $c = r\,D_t s \sin\varepsilon = r\,v \sin\varepsilon$ (57). When $r = r_1$ and $v = V$, let $\varepsilon = \varepsilon_1$; we shall have $c = r_1\,V \sin\varepsilon_1$ (58). That is, when the initial velocity and radius vector are the same, the area described in the unit of time is dependent on the angle which the direction of the initial velocity makes with the radius vector; and since the eccentricity is dependent upon the value of c, (51) it follows, that when r_1, V, and consequently h, are the same, that the eccentricity is dependent upon the same angle, because m is constant for the same body. We also see (53) that the semi-conjugate axis is dependent upon the same angle. But the semi-transverse axis (54) is not dependent on c.

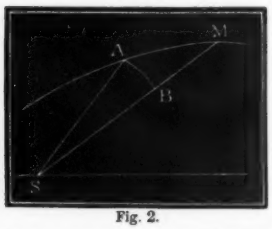

Fig. 2.

19. Since c is twice the area described in the unit of time, in the ellipse it is evidently equal to twice the area of the ellipse divided by the time of revolution in the ellipse, which call T, and we shall have therefore,

$$c = \frac{2\pi ab}{T},\quad c^2 = \frac{4\pi^2 a^2 b^2}{T^2} = \frac{4\pi^2 a^4 (1-e^2)}{T^2}.$$

But from (51) we have semi-parameter $= \frac{c^2}{m} = a(1-e^2)$, and $m = \frac{c^2}{a(1-e^2)}$. $\therefore\ m = \frac{4\pi^2 a^3}{T^2}$ (59). Since m is constant for the same body, we see that the time of revolution depends wholly upon the value of the transverse axis; and if a circle and any number of ellipses be described on the same transverse axis, the time of revolution in all of them will be the same.

20. We have seen, Art 14, that when $h = \frac{m^2}{c^2}$, the curve is a circle. Calling r_1 the radius, we have

$$c^2 = \frac{4\pi^2 r_1^4}{T^2}, \qquad \frac{m}{c^2} = \frac{4\pi^2 r_1^3}{T^2} \times \frac{T^2}{4\pi^2 r_1^4} = \frac{1}{r_1}, \text{ and}$$

$$(60) \qquad \frac{m^2}{c^2} = h = \frac{2m}{r_1} - V^2 = \frac{m}{r_1}, \qquad \text{or } V^2 = \frac{m}{r_1},$$

for the velocity in a circle. Calling V_c the velocity in a circle, and V_p the velocity in a parabola, we shall have, from (56) and (60),

$$(61) \qquad V_p^2 : V_c^2 :: \frac{2m}{r_1} : \frac{m}{r_1} :: 2 : 1. \quad \therefore V_p = V_c\sqrt{2}.$$

That is, *The velocity in a parabola, at any point, is equal to the velocity in a circle passing through the same point, into the square root of* 2, *the centre of the circle being at the focus of the parabola.* We can, therefore, find the velocity in a parabola at any point, by knowing its distance from the sun.

21. We have represented the attractive force by $R = \frac{m}{r^2}$. To interpret this expression, let M equal the attraction of the central body, at the unit of distance, and then, according to NEWTON's law, $\frac{M}{r^2}$ will equal the attraction at the distance r. Let m_1 equal the attraction of the revolving body at the unit of distance, then $\frac{m_1}{r^2}$ will equal the attraction at the same distance r. But both these forces tend to draw the bodies towards each other; and hence, the whole attractive force which causes the bodies to approach each other is

$$(62) \qquad \frac{M}{r^2} + \frac{m_1}{r^2} = \frac{M + m_1}{r^2} = R.$$

This is the force which we have represented by $\frac{m}{r^2}$. Consequently, $M + m_1 = m$ (63). For any other revolving body we have $M + m_2 = m'$.

22. We have already found $m = \frac{4\pi^2 a^3}{T^2}$. For any other body revolving at the mean distance a', we have $m' = \frac{4\pi^2 a'^3}{T'^2}$. Combining these two equations, we have the proportion

$$m\,T^2 : m'\,T'^2 :: a^3 : a'^3. \tag{64}$$

When m and m' are nearly equal, as is the case in the solar system, we may regard them equal, and we shall have

$$T^2 : T'^2 :: a^3 : a'^3. \tag{65}$$

That is, *the squares of the periodic times are to each other as the cubes of the mean distances from the sun.* This is KEPLER'S celebrated third law. From (64) we see that it is not strictly true. We have

$$\frac{a}{a'} = \alpha = \left(\frac{T}{T'}\right)^{\frac{2}{3}} \left(\frac{m}{m'}\right)^{\frac{1}{3}} = \left(\frac{T}{T'}\right)^{\frac{2}{3}} \left(\frac{M+m_1}{M+m_2}\right)^{\frac{1}{3}}.$$

Calling $M = 1$, and $\left(\frac{M+m_1}{M+m_2}\right)^{\frac{1}{3}} = \left(\frac{1+m_1}{1+m_2}\right)^{\frac{1}{3}} = 1 + \frac{m_1 - m_2}{3}$, by developing and retaining only the first powers of m_1 and m_2, the ratio of the mean distances, or

$$\alpha = \left(\frac{T}{T'}\right)^{\frac{2}{3}} \left(1 + \frac{m_1 - m_2}{3}\right). \tag{66}$$

23. When the times of revolution and the mean distances are given, we can find the difference between the masses of the two planets;

for $$M + m_1 = \frac{4\pi^2 a^3}{T^2}, \quad M + m_2 = \frac{4\pi^2 a'^3}{T'^2}, \quad \text{and}$$

$$m_1 - m_2 = 4\pi^2 \left(\frac{a^3}{T^2} - \frac{a'^3}{T'^2}\right), \tag{67}$$

where m_1 and m_2 are the masses according to NEWTON'S law.

24. Equations (59) can be employed to find the mass of a planet which has a satellite. Call t and a_1 the time of revolution and mean distance of the satellite, m, its mass, and m_p the mass of the planet. We obtain

(68) $M + m_p = \frac{4\pi^2 a^3}{T^2}$, $m_p + m_s = \frac{4\pi^2 a_1^3}{t^2}$; and hence

(69) $M + m_p : m_p + m_s :: \frac{a^3}{T^2} : \frac{a_1^3}{t^2} :: \left(\frac{a}{a_1}\right)^3 : \left(\frac{T}{t}\right)^2$.

By subtraction (68) gives

(70) $M - m_s = 4\pi^2 \left(\frac{a^3}{T^2} - \frac{a_1^3}{t^2}\right)$; or $m_s = M - 4\pi^2 \left(\frac{a^3}{T^2} - \frac{a_1^3}{t^2}\right)$.

All the quantities a, a_1, T, t, being derived from observation, m_s is known, which substituted in (69) will give the value of m_p.

ON SPHERICAL ANALYSIS.*

By George Eastwood, Saxonville, Mass.

Proposition I.

To find the equation of a great circle of the sphere, which passes through the origin, and which makes a given angle with the axis of x.

Let OX, OY, be quadrantal arcs of great circles of the sphere intersecting each other in the point O; and let OP be the great circle whose equation is required. Through any point P, pass the great circle arcs XPM, YPN, and designate ON by x, OM by y, OP by r, the angle YOX by ω, and the angle POX by φ. Then, in the spherical triangle XOM, we have

* The following are among the works and memoirs I have had occasion to consult. — *Grundriss der Sphärick Analytischen*, by Professor Gudermann, of Cleve, published about 1830, in which, so far as I know, the idea of expressing the equations of spherical curves in tangent-functions of their rectangular co-ordinate arcs, was first conceived and developed into a system. — Two elaborate memoirs *On the Equations of Loci traced on the Surface of the Sphere*, by the late Professor Davies, of the Royal Military Acade-

$$\cos XM = \cos OM \cos OX + \sin OM \sin OX \cos \angle O,$$
$$= \sin y \cos \omega, \text{ since } OX = \text{a quadrant},$$
$$\cos OM = \cos y = \cos XM \cos XO + \sin XM \sin XO \cos \angle X,$$
$$= \sin XM \cos X,$$
$$= (1 - \sin^2 y \cos^2 \omega)^{\frac{1}{2}} \cos X,$$
$$\cos^2 y = (1 - \sin^2 y + \sin^2 y \sin^2 \omega) \frac{1}{1 + \tan^2 X},$$
$$\tan^2 X = \sin^2 \omega \tan^2 y,$$

(1) $$\tan X = \sin \omega \tan y.$$

For similar reasons, the triangle YON gives

(2) $$\tan Y = \sin \omega \tan x;$$

(3) $$\therefore \frac{\tan y}{\tan x} = \frac{\tan X}{\tan Y}.$$

But the triangle OPX gives

$$\cos PX = \sin r \cos \varphi,$$
$$\cos r = \sin PX \cos X,$$
$$= (1 - \sin^2 r \cos^2 \varphi)^{\frac{1}{2}} \cos X,$$
$$\therefore \frac{1 + \tan^2 X}{1 + \tan^2 r} = 1 - \frac{\tan^2 r}{1 + \tan^2 r} + \frac{\tan^2 r \sin^2 \varphi}{1 + \tan^2 r}.$$

(4) $$\therefore \tan X = \sin \varphi \tan r.$$

Likewise, the triangle OPY gives

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my, Woolwich, England, published in 1832 in the XIIth Volume of the Transactions of the Royal Society of Edinburgh. — "Researches on Spherical Geometry," by the late Professor GILL, published in 1836, in the *Mathematical Miscellany*. — Two Geometrical Memoirs *On the General Properties of the Cone of the Second Degree*, and *on Spherical Conics*, by CHASLES; translated by Rev. CHARLES GRAVES, Fellow and Tutor of Trinity College, Dublin, with an Appendix *On the Application of Analysis to Spherical Geometry*, and published in 1841. — *Essai de Géomètrie Analytique de la Sphère*, by M. BORGNET, published by BACHELIER in 1847. I only know of this work through references to it, all my efforts to procure a copy having been unsuccessful. — Memoir *On Spherical Analysis*, by M. VANNSON, published in TERQUEM and GERONO's *Nouvelles Annales de Mathématiques*, Tome XVII.

(5) $$\tan Y = \sin(\omega - \varphi)\tan r.$$

Hence, by virtue of (3),

(6) $$\tan y = \frac{\sin\varphi}{\sin(\omega - \varphi)}\tan x,$$

is the required equation, which holds for any axes.

If the axes of reference be rectangular, then $\omega = \frac{\pi}{2}$, and (6) reduces to

(7) $$\tan y = \tan\varphi \tan x.$$

Cor. 1. In a plane, the equation of a straight line passing through the origin, when referred to oblique axes, is

(8) $$y = \frac{\sin\varphi}{\sin(\omega - \varphi)}x,$$

and when referred to rectangular axes it is

(9) $$y = \tan\varphi \,.\, x;$$

therefore (6) and (7) are remarkably elegant types of (8) and (9).

Cor. 2. In the triangles ONY, OMX, we have

$$\tan YNX = \frac{\tan\omega}{\cos x},$$

$$\tan YMX = \frac{\tan\omega}{\cos y},$$

$$\therefore \frac{\cos y}{\cos x} = \frac{\tan N}{\tan M}.$$

Assuming the second member to be constant, this equation will define the position of the point P.

Remark. — In geography and astronomy, when we wish to fix the position of a point upon the earth or in the heavens, we usually refer it to the equator and the first meridian, as axes of reference; and then determine the point from its latitude and longitude, or its declination and right ascension. But, in place of the arc of latitude or of declination, we may also take for ordinate the projection of

this arc upon the first meridian. The latter method of determining the position of a point on the sphere has the advantage of giving symmetrical results with respect to x and y, and of rendering it more easy to compare them with the analogous ones in a plane. In order that we may readily distinguish these two systems of co-ordinates, the one from the other, we propose to call the first system geographical, and the second geometrical co-ordinates. The formula for passing from one system to the other is readily found by the resolution of a right spherical triangle. For, if we call $\bar{y}$ the latitude, and y its projection upon the first meridian, or axis of y, then

$$\tan \bar{y} = \tan y \cos x, \tag{5}$$

in which x denotes the longitude of the proposed point.

There is a particular case where the geometrical co-ordinates will not determine the point. This case occurs when $x = \frac{\pi}{2}$, for then y is also equal to $\frac{\pi}{2}$; and the two arcs perpendicular to the axes, which give, in general, the position of the point at their intersection, confound themselves in this specific case. Under these circumstances it will be necessary to take the latitude of the point to fix its position.

[To be Continued.]

THE LAWS OF FALLING BODIES, ADAPTED TO ELEMENTARY INSTRUCTION.

By Thomas Sherwin, Principal of the English High School, Boston.*

I. When a body moves over equal spaces in equal times, it is said to move *uniformly*, or with *uniform velocity*.

* The writer claims no originality in this article. It is merely a compilation, with some modifications and developments, designed to give the learner a simple yet demonstrative exposition of the principles of uniformly accelerated motion, particularly with respect to falling bodies.

This is the kind of motion which results from a force that acts only for an instant, as a momentary impulse, after which the body is abandoned to itself. Under such circumstances, the body must move uniformly and in a straight line. For, in consequence of inertia, the body, unless acted upon by some new force, can neither increase nor diminish its velocity, nor can it change the direction of its motion.

The *velocity* of a body is the space which that body describes uniformly in a unit of time, as a second.

Let u be the velocity, or the space which the body describes in a second; then tu is evidently the space described in t seconds. Calling this space s, we have

(1) $s = ut$; from which we derive

(2) $t = \frac{s}{u}$, and (3) $u = \frac{s}{t}$.

Hence, in uniform motion,

The space is equal to the product of the time and velocity; the time is equal to the quotient of the space divided by the velocity; and the velocity is equal to the quotient of the space divided by the time.

It is also evident that:

When the velocities of two bodies are the same, the spaces are as the times; and, when the times are the same, the spaces are as the velocities.

II. A body continuously acted upon by a constant force moves with a *uniformly accelerated velocity*. For, a body free to move, and urged by such a force, retains all the velocity that has at any time already been imparted to it, and receives equal increments of velocity in equal increments of time.

Although the force of gravity varies inversely as the square of the distance from the earth's centre, yet, within moderate distances from the earth's surface, this force may be regarded as uniform.

Hence, a body near the surface of the earth, and falling under the influence of gravity, descends with uniformly accelerated velocity.

In the descent of a heavy body, the *accelerating force* is represented by the velocity generated in a unit of time. Taking a second as the unit of time, its measure is the velocity per second generated in a second.

Let t be the time of descent estimated in seconds, and k the same time, estimated in instants so small that, during any one of them, the motion may be considered uniform. Suppose that gravity communicates to the body at the commencement of each instant all the velocity it can impart during that instant. Then, if n is the number of these instants in a second,

$$k = nt. \tag{1}$$

Let i be the space which the accelerating force can make the body pass over in an instant; then, i, $2i$, $3i$..... ni ..... ki, will be the spaces passed over during the successive instants of the time t.

Suppose that, at the end of the first second, the accelerating force ceases to act; the body, which passed over the space ni, in the nth instant, would then move uniformly, and in the succeeding second would pass over the space $n^2 i$. If, therefore, we designate by g the accelerating force, we have

$$g = n^2 i. \tag{2}$$

If at the end of the time t we suppose the action of the accelerating force again to cease, the body which passed over the space ki in the kth instant, would again move uniformly, and, in the succeeding second, would pass over the space nki, and this space measures the velocity acquired at the end of the time t. Calling this velocity v, we have $v = nki$. If in this value of v we put nt instead of k, and in the result, g instead of $n^2 i$, we have

$$v = gt. \tag{3}$$

Hence, *The velocity acquired at the end of any time is equal to the accelerating force multiplied by the time in seconds.*

The whole space passed over is the sum of the arithmetical progression, i, $2i$, $3i$, &c. given above. Calling this space s,

$$s = (i + ki)\frac{k}{2} = (1 + k)\frac{ki}{2}.$$

The instants into which the time t was conceived to be divided were supposed to be so small that, during any one of them, the motion might be regarded as uniform. But as gravity acts continuously, the time during which the velocity can be regarded as uniform must be infinitely small, and therefore the number of instants in the time t must be infinitely great.

Hence k is infinitely great with respect to 1; this latter may, therefore, be omitted, and

(4) $$s = \frac{k^2 i}{2} = \frac{n^2 t^2 i}{2} = \frac{g t^2}{2}.$$ Hence,

The space is equal to half the accelerating force multiplied by the square of the time in seconds.

Let v and v' be the respective velocities acquired in the times t and t'. Then

$$v = g t, \text{ and } v' = g t';$$

$$\therefore v : v' = g t : g t' = t : t'.$$ That is,

The final velocities are to each other as the times.

Let s and s' be the spaces passed over in the times t and t'; then

$$s = \frac{g t^2}{2}, \text{ and } s' = \frac{g t'^2}{2};$$

$$\therefore s : s' = \frac{g}{2} t^2 : \frac{g}{2} t'^2 = t^2 : t'^2 = v^2 : v'^2.$$ That is,

The entire spaces are to each other as the squares of the times, or as the squares of the final velocities.

The space passed over in t seconds being $\frac{g}{2} t^2$, and that passed over in $t - 1$ seconds being $\frac{g}{2}(t-1)^2$, their difference, $\frac{g}{2}(2t - 1)$, desig-

nates the space passed over in any one second. Substituting for t successively, 1, 2, 3, 4, &c., and designating the results by s_1, s_2, s_3, s_4, &c., we have,

$$s_1 = \frac{g}{2} \times 1,\ s_2 = \frac{g}{2} \times 3,\ s_3 = \frac{g}{2} \times 5,\ s_4 = \frac{g}{2} \times 7, \text{ \&c.}$$ Hence,

$$s_1 : s_2 : s_3 : s_4, \text{ \&c.} = 1 : 3 : 5 : 7, \text{ \&c.}$$ That is,

The spaces passed over in the separate successive seconds are as the successive odd numbers, 1, 3, 5, 7, &c.

The equation $s_1 = \frac{1}{2} g$, given above, designates the space passed over in the first second of descent, and gives $2 s_1 = g$. But g is the measure of the accelerating force, or the velocity acquired at the end of the first second.

Hence, *The velocity at the end of the first second is twice the space passed over during that second.*

By experiments with the pendulum at Paris, g has been found equal to 32.18 feet. Other experiments have made it equal to $32\frac{1}{6}$ feet, which may be considered as sufficiently accurate for common use, although the value will vary slightly for different latitudes.

The final velocity, as we have seen, is $g t$. But the whole space passed over is $\frac{1}{2} g t^2$, and the average velocity per second is $\frac{1}{2} g t^2$ divided by t, or $\frac{1}{2} g t$. Hence,

The final velocity per second is always double the average velocity per second for the whole descent.

III. Bringing together the important formulæ, we have,

(1) $v : v' = t : t'$; (2) $s : s' = t^2 : t'^2$;

(3) $s : s' = v^2 : v'^2$; (4) $s_1 : s_2 : s_3 : s_4$, &c. $= 1 : 3 : 5 : 7$, &c.

(5) $v = g t$; $\therefore$ (6) $t = \frac{v}{g}$;

(7) $s = \frac{1}{2} g t^2$; $\therefore$ (8) $t = \sqrt{\frac{2 s}{g}}$.

And by eliminating t from (5) and (7),

(9) $s = \frac{v^2}{2 g}$; $\therefore$ (10) $v = \sqrt{2 g s}$.

IV. If we suppose that the body has the velocity u before being subjected to the accelerating force, we have the following formulæ:

(1) $v = u + gt$; $\therefore$ (2) $t = \frac{v-u}{g}$;

(3) $s = ut + \frac{1}{2}gt^2$; $\therefore$ (4) $t = \frac{-u \pm \sqrt{u^2 + 2gs}}{g}$;

(5) $s = \frac{v^2 - u^2}{2g}$; $\therefore$ (6) $v = \sqrt{u^2 + 2gs}$.

V. If, however, the body is projected vertically upward with the velocity u, in which case v and u are of opposite signs, the formulæ become

(1) $v = u - gt$; $\therefore$ (2) $t = \frac{u-v}{g}$;

(3) $s = ut - \frac{1}{2}gt^2$; $\therefore$ (4) $t = \frac{u \pm \sqrt{u^2 - 2gs}}{g}$;

(5) $s = \frac{u^2 - v^2}{2g}$; $\therefore$ (6) $v = \sqrt{u^2 - 2gs}$.

The time at the end of which the body will cease to rise is found by making $v = 0$ in equation (1), which gives $u = gt$ $\therefore$ $t = \frac{u}{g}$; and the height to which the body will rise is found by making $v = 0$ in equation (5), which gives $s = \frac{u^2}{2g}$. The value of u just found is the same as that of v in section III., and therefore the values of t and s are equal to those given in the same section. Hence,

If a body be projected vertically upward, it will rise to that height from which it would have to fall in order to acquire the velocity of projection, and the times of ascent and descent will be the same.

When the value of t renders that of v or of s negative, the result indicates that at the end of this time the body is descending again, or that it has descended below the point of projection a distance equal to this space.

THE ELEMENTS OF QUATERNIONS.

[Continued from Page 133.]

V. The Distributive and Associative Principles.

30. Let different operations of the same sort be represented by φ, φ', &c. If $(\varphi + \varphi')\,\varphi'' = \varphi\,\varphi'' + \varphi'\,\varphi''$, and $\varphi\,(\varphi' + \varphi'') = \varphi\,\varphi' + \varphi\,\varphi''$, these operations are said to be *distributive*. Also let $\varphi_1 = \varphi\,\varphi'$, and $\varphi_2 = \varphi'\,\varphi''$; then, if $\varphi_1\,\varphi'' = \varphi\,\varphi_2$, these operations are said to be *associative*.*

31. The general conclusion deduced in § 26 applied to the definition in § 8 shows directly, since vectors are special cases of quaternions, that *vectors are distributive*.

32. If the axes of p, q, and r are coplanar, and β the common intersection of the planes of p, q, and r, another line, α, can always be found, such that $r\,\alpha = \beta$. Then, since $(p + q)\,\beta = p\,\beta + q\,\beta$, we have

(38) $(p+q)r\alpha = pr\alpha + qr\alpha = (pr + qr)\alpha$, or $(p+q)r = pr + qr$;

whence, by (8) and (9),

$$\mathrm{K}(p + q)r = \mathrm{K}r\,.\,\mathrm{K}(p + q) = \mathrm{K}r\,(\mathrm{K}p + \mathrm{K}q),$$
$$\mathrm{K}(p + q)r = \mathrm{K}pr + \mathrm{K}qr = \mathrm{K}r\,.\,\mathrm{K}p + \mathrm{K}r\,.\,\mathrm{K}q.$$

Equating the last members of these equations, we have in general, since the conjugate of a quaternion is a quaternion,

(38 *a*) $\mathrm{K}r(\mathrm{K}p + \mathrm{K}q) = \mathrm{K}r\,.\,\mathrm{K}p + \mathrm{K}r\,.\,\mathrm{K}q$, $r(p+q) = rp + rq$.

33. The three quaternions $\mathrm{S}q$, $\mathrm{V}q$, and r, satisfy the condition of coplanarity of axes, since, by § 19, Ax. $\mathrm{S}q$ may be taken in any direction we please. Therefore, (38) and (38 *a*) give for any two quaternions

---

* With greater exactness this may be called the associative principle *of multiplication*, and the equation $(\varphi + \varphi') + \varphi'' = \varphi + (\varphi' + \varphi'')$ the expression of the associative principle *of addition*. Similarly the equation $\varphi + \varphi' = \varphi' + \varphi$ may be called the expression of *the commutative principle of addition*. We see directly in § 8 that quaternions satisfy both these latter equations.

(39) $qr=(\mathrm{S}q+\mathrm{V}q)r=\mathrm{S}q.r+\mathrm{V}q.r=\mathrm{S}q.\mathrm{S}r+\mathrm{S}q.\mathrm{V}r+\mathrm{V}q.\mathrm{S}r+\mathrm{V}q.\mathrm{V}r,$

which is equivalent to the two equations

(39′) $\mathrm{S}qr=\mathrm{S}q.\mathrm{S}r+\mathrm{S}(\mathrm{V}q.\mathrm{V}r),\quad \mathrm{V}qr=\mathrm{S}q.\mathrm{V}r+\mathrm{S}r.\mathrm{V}q+\mathrm{V}(\mathrm{V}q.\mathrm{V}r).$

34. As $p+q$ is a quaternion it may be substituted for q in (39), then in general for any three quaternions

(40) $$(p+q)r=\mathrm{S}(p+q)\mathrm{S}r+\mathrm{S}(p+q)\mathrm{V}r+\mathrm{V}(p+q)\mathrm{S}r+\mathrm{V}(p+q)\mathrm{V}r$$
$$=(\mathrm{S}p+\mathrm{S}q)\mathrm{S}r+(\mathrm{S}p+\mathrm{S}q)\mathrm{V}r+(\mathrm{V}p+\mathrm{V}q)\mathrm{S}r+(\mathrm{V}p+\mathrm{V}q)\mathrm{V}r;$$

whence, as the first three terms of the last member of (40) satisfy the condition of coplanarity of axes, we have, by §§ 32 and 31 and (39),

(41) $$(p+q)r=\mathrm{S}p.\mathrm{S}r+\mathrm{S}q.\mathrm{S}r+\mathrm{S}p.\mathrm{V}r+\mathrm{S}q.\mathrm{V}r+\mathrm{V}p.\mathrm{S}r+\mathrm{V}q.\mathrm{S}r+\mathrm{V}p.\mathrm{V}r+\mathrm{V}q.\mathrm{V}r$$
$$=pr+qr.$$

By the same process used in obtaining (38*a*) from (38), we obtain

(41*a*) $$r(p+q)=rp+rq.$$

Equations (41) and (41*a*) show that in general *quaternions are distributive*. By an easy extension we have for any number of quaternions,

(42) $(p+p'+p''+\&c.)\,(q+q'+q''+\&c.)=pq+pq'+\&c.+p'q+p'q'+\&c.$

Equation (42) is merely a more general expression of *the distributive principle*.

35. Let p and q be coplanar, and β the intersection of their common plane with that of r; then another line, α, can always be found in the plane of r, and two other lines, γ and δ, in the common plane of p and q, such that $r\alpha=\beta$, $q\beta=\gamma$, and $p\gamma=\delta$. Denoting the product qr by q', and pq by p', we have $q'\alpha=\gamma$, and $p'\beta=\delta$; whence

$$pq'=\delta\div\alpha=p'r.$$

This equation may also be written

(43) $$p(qr)=(pq)r,\quad \text{or simply } p\,.\,qr=pq\,.\,r.$$

Taking the conjugates of both members of (43) gives

$$(43a)\quad \mathrm{K}qr\,.\,\mathrm{K}p = \mathrm{K}r\,.\,\mathrm{K}pq = \mathrm{K}r\,\mathrm{K}q\,.\,\mathrm{K}p = \mathrm{K}r\,.\,\mathrm{K}q\,\mathrm{K}p;$$

but, as these conjugates are quaternions subject only to the condition that the *last two* written shall be coplanar, we see that (43) is true when only q and r, as well as when only p and q, are coplanar.

36. Let α and β be two vectors, perpendicular to each other, then, as a special case of (26), their product, α_1, will be perpendicular to both α and β, and such that we shall have the three symmetric equations,

$$(44)\quad \mathrm{U}\alpha\,.\,\mathrm{U}\beta = \mathrm{U}\alpha_1,\quad \mathrm{U}\beta\,.\,\mathrm{U}\alpha_1 = \mathrm{U}\alpha,\quad \mathrm{U}\alpha_1\,.\,\mathrm{U}\alpha = \mathrm{U}\beta.$$

Let γ be any other vector, and let $q = \beta\gamma$. Then, by (31), $\mathrm{U}\alpha_1\gamma$ will represent the operation of turning a line from the direction of γ^{-1} to that of α_1, or, what is the same thing, from that of γ to that of $-\alpha_1$. But also, in the same way, $\mathrm{U}q$ and $\mathrm{U}\alpha$ represent the respective operations of turning lines from the direction of γ to that of $-\beta$, and from that of $-\beta$ to that of $-\alpha_1$. Therefore,

$$\mathrm{U}\alpha\,.\,\mathrm{U}q = \mathrm{U}\alpha q = \mathrm{U}\alpha_1\gamma;$$

and since evidently $\mathrm{T}\alpha q = \mathrm{T}\alpha_1\gamma$, it follows that

$$(45)\qquad \alpha q = \alpha_1\gamma, \qquad \text{or } \alpha\,.\,\beta\gamma = \alpha\beta\,.\,\gamma.$$

37. Of any three vectors, α, β, and γ, let one, as α, be expressed, by § 22, as the sum of two other vectors, α_1 and α_2, whose directions are respectively parallel and perpendicular to that of β. Then, as a special case of (43), $\alpha_1\,.\,\beta\gamma = \alpha_1\beta\,.\,\gamma$, and by (45), $\alpha_2\,.\,\beta\gamma = \alpha_2\beta\,.\,\gamma$; whence, by (41), for any three vectors,

$$(46)\quad \alpha\,.\,\beta\gamma = (\alpha_1+\alpha_2)\,.\,\beta\gamma = \alpha_1\,.\,\beta\gamma + \alpha_2\,.\,\beta\gamma = \alpha_1\beta\,.\,\gamma + \alpha_2\beta\,.\,\gamma = (\alpha_1+\alpha_2)\beta\,.\,\gamma = \alpha\beta\,.\,\gamma.$$

38. By (42), we develop the following equation:

$$\begin{aligned} p\,.\,qr &= (\mathrm{S}p + \mathrm{V}p)\,.\,(\mathrm{S}q + \mathrm{V}q)\,(\mathrm{S}r + \mathrm{V}r) \\ (47)\qquad &= \mathrm{S}p\,.\,\mathrm{S}q\,\mathrm{S}r + \mathrm{S}p\,.\,\mathrm{S}q\,\mathrm{V}r + \mathrm{S}p\,.\,\mathrm{V}q\,\mathrm{S}r + \mathrm{S}p\,.\,\mathrm{V}q\,\mathrm{V}r \\ &\quad + \mathrm{V}p\,.\,\mathrm{S}q\,\mathrm{S}r + \mathrm{V}p\,.\,\mathrm{S}q\,\mathrm{V}r + \mathrm{V}p\,.\,\mathrm{V}q\,\mathrm{S}r + \mathrm{V}p\,.\,\mathrm{V}q\,\mathrm{V}r. \end{aligned}$$

Since, by § 19, the plane of a given scalar may be considered as coinciding with any plane we please in space, all the terms in the last member of (47) except the last, satisfy one or the other of the conditions of § 35; whence, by (43), (46), and (47),

$$
\begin{aligned}
p\,.\,qr &= \mathrm{S}p\,\mathrm{S}q\,.\,\mathrm{S}r + \mathrm{S}p\,\mathrm{S}q\,.\,\mathrm{V}r + \mathrm{S}p\,\mathrm{V}q\,.\,\mathrm{S}r + \mathrm{S}p\,\mathrm{V}q\,.\,\mathrm{V}r \\
&\quad + \mathrm{V}p\,\mathrm{S}q\,.\,\mathrm{S}r + \mathrm{V}p\,\mathrm{S}q\,.\,\mathrm{V}r + \mathrm{V}p\,\mathrm{V}q\,.\,\mathrm{S}r + \mathrm{V}p\,\mathrm{V}q\,.\,\mathrm{V}r \qquad (48) \\
&= (\mathrm{S}p + \mathrm{V}p)\,(\mathrm{S}q + \mathrm{V}q)\,.\,(\mathrm{S}r + \mathrm{V}r) = pq\,.\,r.
\end{aligned}
$$

Equation (48) shows that in general, *quaternions are associative.*

Mathematical Monthly Notices.

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Tables of Victoria. Computed with Regard to the Perturbations of Jupiter and Saturn. By F. BRÜNNOW, Ph. Dr., Professor of Astronomy in the University of Michigan, and Director of the Observatory at Ann Arbor. Printed by order of the Board of Regents. New York: B. Westermann & Co. London: N. Trübner & Co. 1857. 4to. pp. 77.

These Tables embody the results of the author's labors upon the asteroid Victoria (sometimes called Clio) since its discovery, September 13, 1850; and are based upon the observations made at the oppositions in 1850, 1852, 1853, 1854, 1856, and 1857. The elements derived from the first five of these oppositions, and the numerical expression of the perturbations produced by Jupiter and Saturn, may be found in the *Astronomische Nachrichten*, No. 1077; and the corrections of these elements, as given by the observations of the opposition of 1857, in the *Astronomical Journal*, No. 108. The smallness of these corrections showed that the resulting elements were sufficiently accurate to form the basis of a set of Tables which are now before us. These Tables, by means of 37 different arguments, give the perturbations of M, the mean anomaly, of $\log r$, the logarithm of the radius vector, and of ζ, the perpendicular to the plane of the asteroid's orbit. These are HANSEN'S co-ordinates, and the general expressions for their perturbations are found in the *Astronomische Nachrichten*, No. 799. Having found these perturbed co-ordinates, we readily pass to the rectangular co-ordinates, referred to the plane of the equator, and the line of the equinoxes. Abbreviated tables are given for finding an approximate place when only five principal terms of the perturbations are used, by means of which an ephemeris sufficiently accurate for all the purposes of observation can be rapidly computed. When it is remembered that an accurate ephemeris is only needed for a month or so before and after the opposition, while a rough one for the purpose of identification is needed for all the rest of the year, the advantage of this feature will be apparent. The Tables are well arranged for use, and printed in fine style. The author has placed astronomers, as well as the science, under renewed obligations by the publication of this timely and valuable work.

Editorial Items.

THE following gentlemen have sent us solutions of the Prize Problems in the November Number, Vol. II., of the Monthly: —

JAMES F. ROBERSON, Senior Class, Indiana University, Bloomington, answered Problems III. and IV.

GEORGE B. HICKS, Cleveland, Ohio, answered Problems III., IV., and V.

WILLIAM HINCHCLIFFE, Barre Plains, Mass., answered Problems III. and IV.

D. M. HUDSON, Paris, Jennings Co., Indiana, answered Problems I. and II.

DAVID TROWBRIDGE, Perry City, Schuyler Co., N. Y., answered Problems III., IV., and the first part of V.

HENRY E. PRINDLE, Ashtabula, Ohio, answered Problems III. and IV.

We would also acknowledge valuable solutions from ASAPH HALL, Esq., Assistant at the Observatory of Harvard College, and JAMES CLARK, Esq., Wayne, Me.

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